MCV 4U SCALAR MULTIPLICATION OF VECTORS

INVESTIGATION #1

Complete the following investigation in the space provided below.

1. Draw any vector $\vec{a}$, $|\vec{a}| = 3$ cm
2. Draw $\vec{u} = \vec{a} + \vec{a}$. $(\vec{u} = 2\vec{a})$

   What is the magnitude of $\vec{u}$?
3. Draw $\vec{v} = \vec{a} + \vec{a} + \vec{a} + \vec{a}$. $(\vec{v} = 4\vec{a})$

   What is the magnitude of $\vec{v}$?
4. Draw $\vec{w} = -\vec{a}$.

   What is the magnitude of $\vec{w}$?
5. Draw $\vec{z} = -\vec{a} - \vec{a} - \vec{a}$. $(\vec{z} = -3\vec{a})$

   What is the magnitude of $\vec{z}$?

- Are there any geometric similarities among the vectors $\vec{u}, \vec{v}, \vec{w}$, and $\vec{z}$? → parallel "collinear"

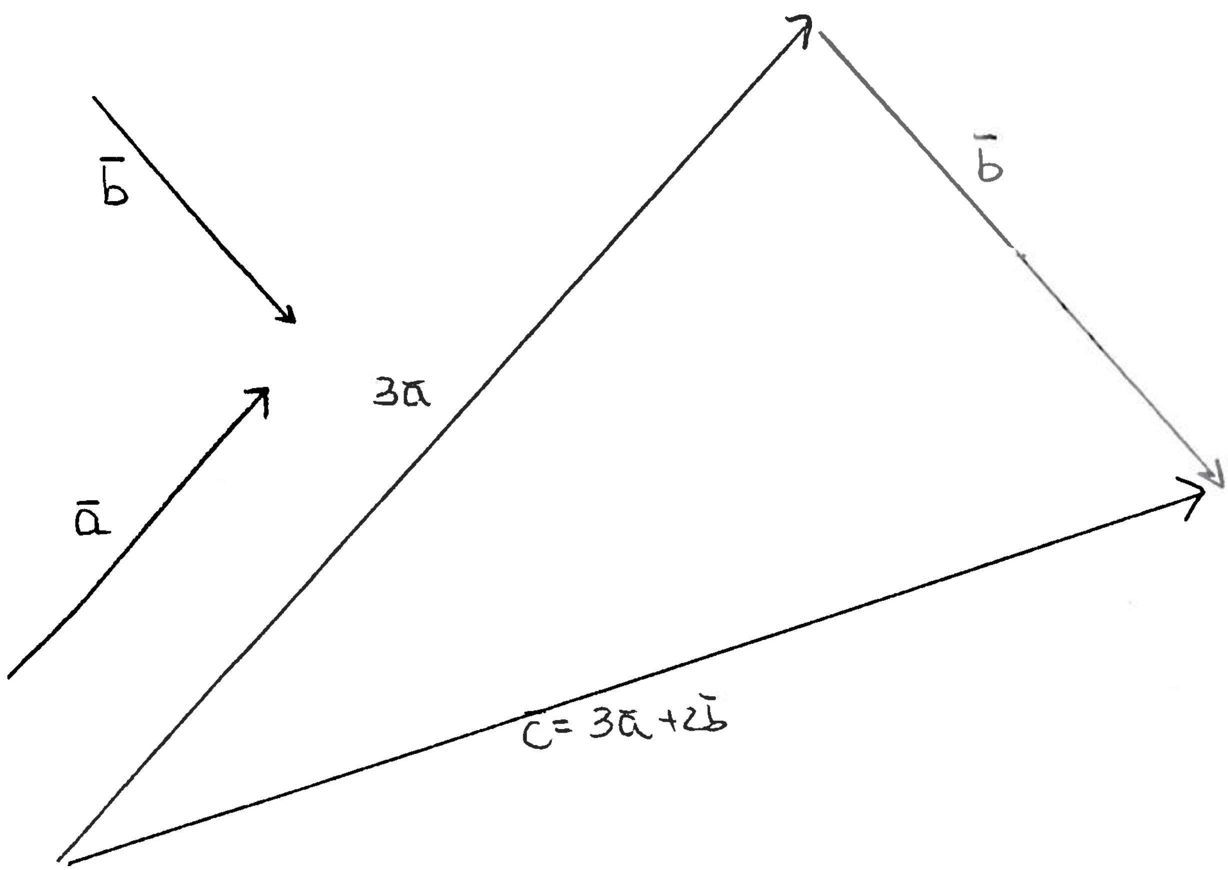
- when 2 vectors are parallel or lie on the same straight line, they are described as being COLLINEAR.
- lines can be translated so that they lie in the same straight line.
- vectors that are not parallel are not collinear.
- two vectors are collinear if they are scalar multiples of each other

- If $\vec{v}$ is any vector, and $k \in R$, then $k\vec{v}$ is a SCALAR MULTIPLE of $\bar{v}$.

$\bar{v}$, $k\bar{v}$ are collinear → if $k>0$, $\bar{v}$, $k\bar{v}$ same direction
if $k<0$, $\bar{v}$, $k\bar{v}$ opposite direction
if $k=0$, $k\bar{v}$ is a zero vector

INVESTIGATION #2

Given the vectors $\vec{a}$ and $\vec{b}$ as shown below. Draw the vector $3\vec{a}+2\vec{b}$.

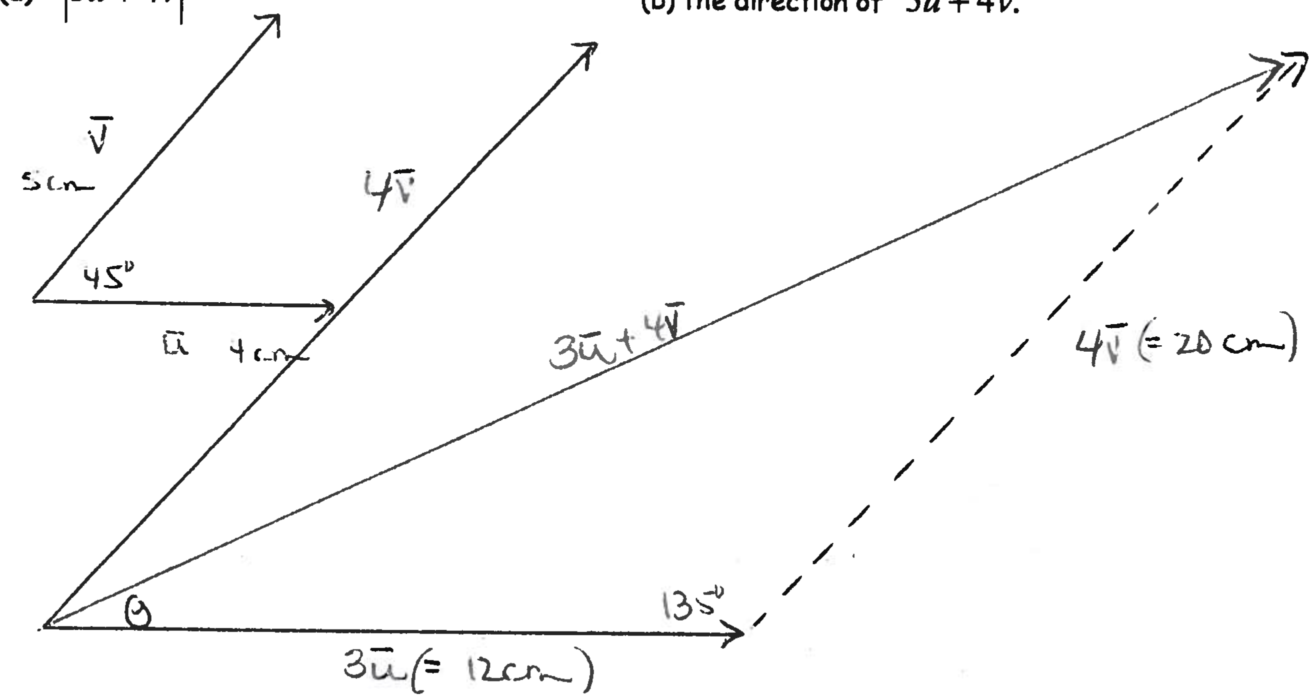


- Any vector $\vec{c}$ in the plane can be expressed as a "LINEAR COMBINATION" of two non-collinear vectors $\vec{a}$ and $\vec{b}$. (where $\vec{a},\vec{b},\vec{c}$ share the same tail).
- $\vec{c} = 3\bar{a} + 2\bar{b}$
- $\vec{c} = s\bar{a} + t\bar{b}$

Example 1

Two non-collinear vectors $\vec{u}$ and $\vec{v}$, with lengths 4 cm and 5 cm, respectively, have an angle of 45° between them. Find:

(a) $|3\vec{u}+4\vec{v}|$ (b) the direction of $3\vec{u}+4\vec{v}$.



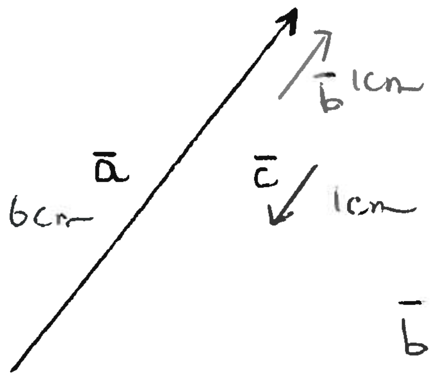
$$|3\bar{u}+4\bar{v}|^2 = 12^2 + 20^2 - 2(12)(20)\cos 135^\circ$$
$$= 544 + 339.41$$
$$= 883.41$$
$$\therefore |3\bar{u}+4\bar{v}| = 29.7\text{ cm}$$
$$\doteq 30\text{ cm}$$

$$\frac{\sin\theta}{20} = \frac{\sin 135^\circ}{29.7}$$
$$\sin\theta = \frac{20\sin 135^\circ}{29.7}$$
$$\theta \doteq 28.4^\circ$$

$\therefore$ magnitude of $3\bar{u}+4\bar{v}$ is 29.7 cm 28.4° to vector $3\bar{u}$.

INVESTIGATION #3

Given the vector $\vec{a}$ as shown below. Draw a vector that is parallel to $\vec{a}$ having a length of 1 cm. Draw a second vector that is parallel and opposite to $\vec{a}$ that also has a length of 1.



$\bar{b} = \frac{1}{6}\bar{a} \longrightarrow \bar{b} = \frac{1}{|\bar{a}|}\bar{a}$

$\bar{c} = -\frac{1}{6}\bar{a} \longrightarrow \bar{c} = -\frac{1}{|\bar{a}|}\bar{a}$

$\bar{b}, \bar{c}$ are called UNIT VECTORS!

$|\bar{b}| = 1 = |\bar{c}|$

Example 2

Using the information in example #1, determine the unit vector in the same direction as $3\vec{u} + 4\vec{v}$.

unit vector in same direction as $3\bar{u} + 4\bar{v}$

$= \frac{1}{|3\bar{u} + 4\bar{v}|}(3\bar{u} + 4\bar{v})$

$= \frac{1}{30}(3\bar{u} + 4\bar{v}) = \frac{\bar{u}}{10} + \frac{2\bar{v}}{15}$

Practice Problems

1. Given the following geometric figure. $\overrightarrow{CE} = 6\overrightarrow{AB}$.

   Express each vector in terms of $\vec{a}$ and/or $\vec{b}$.

   $(a)\overrightarrow{BC} = \bar{b}$

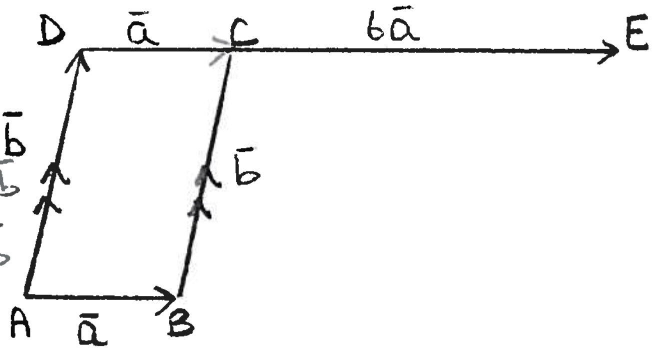
   $(b)\overrightarrow{CD} = -\bar{a}$

   $(c)\overrightarrow{AE} = 7\bar{a} + \bar{b}$

   $(d)\overrightarrow{DB} = \bar{a} - \bar{b}$

   $(e)\overrightarrow{ED} = -7\bar{a}$

   $(f)\overrightarrow{AC} = \bar{a} + \bar{b}$



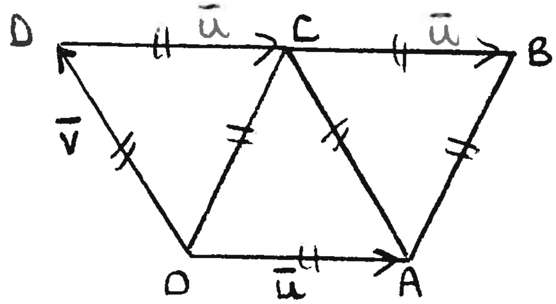
2. In the diagram, $\Delta DOC, \Delta OCA,$ and ΔCAB are equilateral. $\overrightarrow{OA} = \vec{u}$ and $\overrightarrow{OD} = \vec{v}$. Express each of the following as a linear combination of $\vec{u}$ and $\vec{v}$.

(a) $\overrightarrow{OC}$

(b) $\overrightarrow{AB}$

(c) $\overrightarrow{OB}$

(d) $\overrightarrow{AD}$



(a) $\overline{OC} = \bar{u} + \bar{v}$

(d) $\overline{AD} = -\bar{u} + \bar{v}$

(b) $\overline{AB} = -\bar{u} + \bar{v} + \bar{u} + \bar{u} = \bar{u} + \bar{v}$

(c) $\overline{OB} = \bar{v} + 2\bar{u}$

3. Three vectors are non-collinear such that $\vec{p} = \frac{1}{3}\vec{q}$ and $\vec{p} = \frac{2}{5}\vec{r}$.

Determine all integer values of m and n so that $m\vec{q} + n\vec{r} = \vec{0}$.

$\bar{q} = 3\bar{p}$ and $\bar{r} = \frac{5}{2}\bar{p}$

$\therefore m\bar{q} + n\bar{r} = \bar{0}$

$m(3\bar{p}) + (\frac{5}{2}\bar{p})n = \bar{0}$

$(3m + \frac{5}{2}n)\bar{p} = \bar{0}$

$\therefore 3m + \frac{5}{2}n = 0$

$3m = -\frac{5}{2}n$

$6m = -5n$

$\therefore m = 5, n = -6$

$m = 10, n = -12$

$m = -5, n = 6$

(m, n) can be any multiple of $(5, -6)$

Homework

pg. 299-301

#2, 4, 5, 8-10, 12-14, 17, 18